CE-080 Natural Gas Pipeline Flow Calculations

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Natural Gas Pipeline Flow Calculations

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COURSE CONTENT

1. Introduction

There are numerous equations that can be used to make natural gas pipeline flow calculations depending upon various factors, such as the magnitude of the pressure drop, the pipe diameter, the length of the pipeline, the Reynolds number, and whether the flow can be considered isothermal or adiabatic. This course will begin with a discussion of the gas properties needed for the calculations. There will then be a presentation of the various pipeline flow equations and a brief identification of the type of flow for which each is appropriate. Then there will be as section for each of the equations, giving the detailed equation and description of the parameters, along with example calculations of parameters such as flow rate, required diameter or pressure drop.

Image Credit: Wikimedia Commons, Glen Dillon, View of Dampier to Bunbury Natural Gas Pipeline.

2. Learning Objectives

At the conclusion of this course, the student will

- Be familiar with the natural gas properties, density, specific gravity, molecular weight, compressibility factor, and viscosity, and their use in pipeline flow calculations.

- Be able to calculate the compressibility factor for natural gas with specified average gas pressure and temperature and known specific gravity.

- Be able to calculate the viscosity of natural gas with specified average gas pressure and temperature and known specific gravity.

- Be able to obtain a value for the friction factor using the Moody diagram for given Re and ε/D.

- Be able to calculate a value for the friction factor for specified Re and ε/D, using the appropriate equation for f.

- Be familiar with the guidelines for when it is appropriate to use the Darcy Weisbach equation for natural gas pipeline flow calculations.

- Be able to use the Darcy Weisbach equation and the Moody friction factor equations to calculate the frictional pressure drop for a given flow rate of a specified fluid through a pipe with known diameter, length and roughness.

- Be able to use the Weymouth equation to calculate gas flow rate through a pipe with known diameter and length, elevation difference between pipeline inlet and outlet, specified inlet and outlet pressure and enough information to calculate gas properties.

- Be able to use the Panhandle A equation to calculate gas flow rate through a pipe with known diameter and length, elevation difference between pipeline inlet and outlet, specified inlet and outlet pressure and enough information to calculate gas properties.

- Be able to use the Panhandle B equation to calculate gas flow rate through a pipe with known diameter and length, elevation difference between pipeline inlet and outlet, specified inlet and outlet pressure and enough information to calculate gas properties.

3. Topics Covered in this Course

I. Natural Gas Properties

II. Laminar and Turbulent Flow in Pipes

III. Options for Natural Gas Pipeline Flow Calculations

IV. The Darcy Weisbach Equation

V. The Weymouth Equation

VI. The Panhandle A Equation

VII. The Panhandle B Equation

VIII. Example Calculations

IX. Summary

X. References

4. Natural Gas Properties

Natural gas properties that are used in pipeline flow calculations include density, specific gravity, molecular weight, compressibility factor, and viscosity. Each of these natural gas properties will now be discussed briefly.

Density - The density of a gas is its mass per unit volume, typically expressed in lbm/ft^3 or $slugs/ft^3$ in U.S. units and in kg/m^3 for S.I. units. The gas density is an important parameter for gas pipe flow calculations. The density of any gas increases with increasing pressure and decreases with increasing temperature. For relatively short lengths of pipe with small pressure drop, the density will not change throughout the pipe and an incompressible flow equation, like the Darcy Weisbach equation can be used. For long pipelines with large pressure differences from inlet to outlet, however, the density will change appreciably and a calculation approach that takes the changing density of the gas into account must be used.

Two other parameters related to the density are specific volume and specific weight. The specific volume is the inverse of density, typically expressed in ft^3/lbm or $ft^3/slug$ in U.S. units or m^3/kg in S.I. units. The specific weight of a gas is the weight per unit volume, typically expressed in lbf/ft^3 in U.S. units or kN/m^3 in S.I. units. In equation form:

$$\text{specific volume of a gas} = \mathbf{v_{gas} = 1/\rho_{gas}}$$

$$\text{specific weight of a gas} = \boldsymbol{\gamma_{gas} = (\rho_{gas})(g)}$$

Note that g is the acceleration due to gravity (32.17 ft/sec^2 or 9.81 m/s^2)

Specific Gravity - Specific gravity is often a known, specified parameter for natural gas. The specific gravity is the ratio of the density of the gas to the density of air at the same temperature and pressure. Thus, the density of the gas can be calculated from known specific gravity using the equation:

$$\boldsymbol{\rho_{gas} = (G_{gas})(\rho_{air})} \qquad (1)$$

where: $\mathbf{G_{gas}}$ is the specific gravity of a gas

ρ_{air} is the density of air at a specified temperature and pressure

ρ_{gas} is the density of the gas at the same temperature and pressure

Example #1: What is the density of a sample of natural gas of specific gravity 0.65 at standard temperature and pressure (60 ^{o}F and 14.7 psi)? Note that the density of air at 60°F and 14.7 psi is 0.0764 lbm/ft^3.

Solution: The gas density can be calculated with equation (1).

$$\rho_{gas} = (0.65)(0.0764) = \underline{\mathbf{0.050\ lbm/ft^3}}$$

Molecular Weight - The molecular weight of a gas is the mass per mole. The value is the same in any of the commonly used units, lbm/lbmole, g/mole, or kg/kgmole. The molecular weight and specific gravity of a natural gas sample are related to each other by the equation:

$$\mathbf{MW_{gas} = (G_{gas})(MW_{air})} \qquad (2)$$

where: $\mathbf{MW_{gas}}$ is the molecular weight of the natural gas

$\mathbf{MW_{air}}$ is the molecular weight of air **(typical taken to be 28.97)**

$\mathbf{G_{gas}}$ is the specific gravity of the natural gas

Example #2: What is the molecular weight of the natural gas described in Example #1, which has a specific gravity of 0.65?

Solution: Substituting into Equation (2) gives:

$$\mathbf{MW_{gas}} = (0.65)(28.97) = \underline{\mathbf{18.8}}$$

Average Pipeline Pressure - Average pipeline pressure is needed for some natural gas pipeline flow calculations. This average pressure is typically calculated with the following equation:

$$\mathbf{P_{ave} = (2/3)[(P_1^3 - P_2^3)/(P_1^2 - P_2^2)]} \qquad (3)$$

where: $\mathbf{P_1}$ is the pipeline inlet pressure, and

$\mathbf{P_2}$ is the pipeline outlet pressure

This equation gives a value for the average pipeline pressure that is slightly higher than the arithmetic average of the inlet and outlet pressures, as illustrated in the following example.

Example #3: Consider gas flowing through a section of pipeline with an inlet pressure of 1000 psig and outlet pressure of 800 psig. Calculate the arithmetic average of the inlet and outlet pressures and calculate the average pipeline pressure using Equation (3).

Solution: The arithmetic average pressure is:

$\mathbf{P_{ArithAvg}} = (1000 + 800)/2 =$ **<u>900 psig</u>**

Using Equation (3):

$\mathbf{P_{ave}} = (2/3)[(1000^3 - 800^3)/(1000^2 - 800^2)] =$ **<u>903.7 psig</u>**

Compressibility Factor - The compressibility factor of a gas (Z) at any specified temperature and pressure is a measure of the extent of its deviation from ideal gas law behavior at that temperature and pressure. At temperatures much greater than the critical temperature of a gas and/or pressures much less than the critical pressure of a gas, it will follow the ideal gas law and the compressibility factor will be one. If the temperature is low enough and/or the pressure is high enough so that the gas will not exhibit ideal gas behavior, then the value of the compressibility factor will be less than one.

The ideal gas law, as modified by inclusion of the compressibility to allow calculations for non-ideal gases is as follows:

$$\mathbf{PV = ZnRT} \qquad (4)$$

Where: **P** = the absolute pressure of the gas in psia

V = the volume of the gas in ft^3

Z = the compressibility factor of the gas at the specified temperature and pressure

n = the lbmoles of the gas

R = the ideal gas law constant = 10.73 psia-ft^3/lbmole-^oR

T = the absolute temperature of the gas in ^oR

Noting that the number of lbmoles of gas is given by the mass of the gas divided by its molecular weight, we can substitute **n = m/MW** into the ideal gas law equation and solve for the gas density **(ρ = m/V)** to give:

$$\rho = \mathbf{P(MW)/ZRT} \qquad (5)$$

This equation can be used to calculate the density of a sample of natural gas at specified temperature and pressure if its molecular weight and compressibility factor are known.

There are several graphs and equations for compressibility factor as a function of temperature and pressure for gases in general and for natural gas in particular. One that is rather straightforward and easy to use is the CNGA (California Natural Gas Association) equation for the compressibility factor of natural gas:

$$Z = \frac{1}{1 + \dfrac{P_{avg}\, 344400(10)^{1.785G}}{T_f^{\,3.825}}} \qquad (6)$$

Where: **Z** is the compressibility factor, which is dimensionless

$\mathbf{P_{avg}}$ is the average gas pressure in psig

$\mathbf{T_f}$ is the average gas temperature in ^oR

G is the specific gravity of the gas

This equation may be used for average natural gas pressure greater than 100 psig. The compressibility factor is typically taken to be equal to one, if the average gas pressure is less than 100 psig.

Example #4: Determine the compressibility factor of the natural gas of Examples #1 and #2, which has specific gravity equal to 0.65, when it is in a pipeline with average pressure equal to 500 psig and temperature equal to 80 ^oF.

Solution: The 80 ^oF temperature is equal to 80 + 460 ^oR = 540 ^oR. Substituting values for $\mathbf{P_{avg}}$, **G**, and $\mathbf{T_f}$ into Equation (6) gives:

$$\mathbf{Z = 1/\{1 + [(500)(344400)(10^{1.785*0.65})/540^{3.825}]\}}$$

$$\underline{\mathbf{Z = 0.919}}$$

Viscosity - The viscosity of a fluid is a measure of its resistance to flow, and hence is an important parameter in pipe flow calculations. As a measure of resistance to flow, the viscosity of molasses, for example, is very high. The viscosity of water is much lower than that of a "thick" liquid like molasses, and the viscosities of gases are, in general, much lower than the viscosities of liquids. Two different forms of viscosity are sometimes used. The first is dynamic viscosity, typically represented by the Greek letter, μ, and the other is kinematic viscosity, typically represented by the Greek letter, ν. Typical units for dynamic viscosity are Pa-s, centipoise (cp), or lbf-sec/ft^2. For kinematic viscosity, the units are ft^2/sec, m^2/s or centistokes. The relationship between dynamic viscosity and kinematic viscosity is:

$$\nu = \mu/\rho \qquad (7)$$

A convenient equation for calculating the viscosity of natural gas is that of Lee, Gonzalez and Eakin

$$\mu_g = 1 \times 10^{-4} k_v \exp\left\{ x_v \left[\frac{\rho_g}{62.4} \right]^{y_v} \right\} \tag{8}$$

where: $$k_v = \frac{(9.4 + 0.02\, MW_g)T^{1.5}}{209 + 19\, MW_g + T}$$

$$y_v = 2.4 - 0.2\, x_v$$

$$x_v = 3.5 + \frac{986}{T} + 0.01\, MW_g$$

In these expressions, **T** is the natural gas temperature in °R, ρ_g is the density of the gas in lbm/ft^3, $\mathbf{MW_g}$ is the molecular weight of the gas, and μ_g is the dynamic viscosity of the gas in centipoise (cp).

Example #5: Calculate the density and viscosity of the natural gas described in Example #3, which has specific gravity equal to 0.65 and is in a pipeline with an average pressure of 500 psia and temperature of 80 °F.

Solution: From Example #2, the molecular weight of the gas is $\mathbf{MW_g}$ = 18.8. From Example #4, the compressibility factor of the gas is **Z** = 0.919. The density of the gas can now be calculated using Equation #4 as follows:

$$\rho_g = \mathbf{P(MW)/ZRT} = (500)(18.8)/[(0.919)(10.73)(80 + 460)]$$

$$\underline{\rho_g = \mathbf{1.765\ lbm/ft^3}}$$

The viscosity of the gas can now be calculated by first calculating values for k_v, x_v, and y_v, and substituting those values into Equation (8), as follows:

$$\mathbf{k_v} = (9.4 + 0.02*18.8)(540^{1.5})/(209 + 19*18.8 + 540) = 110.9$$

$$\mathbf{x_v} = 3.5 + (986/540) + (0.01*18.8) = 5.514$$

$$\mathbf{y_v} = 2.4 - (0.2*5.514) = 1.297$$

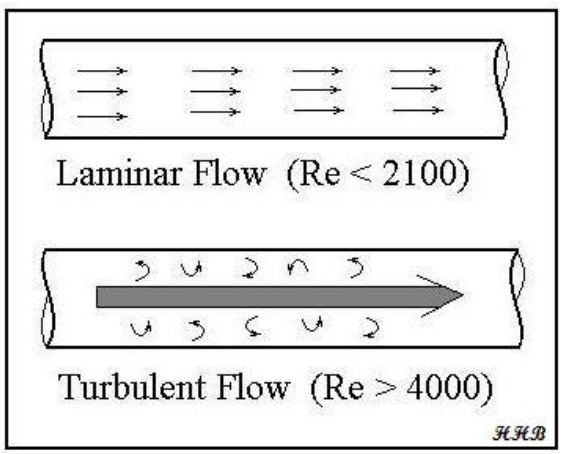
Substituting values into Equation (4) gives:

$$\boldsymbol{\mu_g} = (1/10^4)(110.9)\exp[5.514(1.85/62.4)^{1.297}] = \underline{\mathbf{0.01175\ cp}}$$

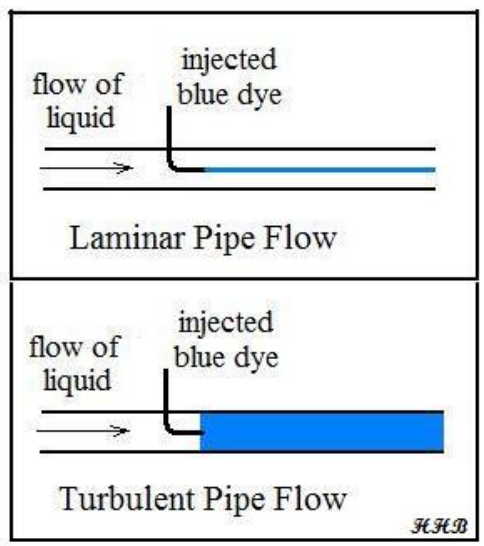
5. Laminar and Turbulent Flow in Pipes

It is often useful to be able to determine whether a given pipe flow is laminar or turbulent. This is necessary, because different methods of analysis or different equations are sometimes needed for the two different flow regimes.

Laminar flow takes place for flow situations with low fluid velocity and high fluid viscosity. In laminar flow, all of the fluid velocity vectors line up in the direction of flow. Turbulent flow, on the other hand, is characterized by turbulence and mixing in the flow. It has point velocity vectors in all directions, but the overall flow is in one direction. Turbulent flow takes place in flow situations with high fluid velocity and low fluid viscosity. The figures below illustrate differences between laminar and turbulent flow in a pipe.



Laminar and Turbulent Pipe Flow



The figure at the right above illustrates the classic experiments of Osborne Reynolds, in which he injected dye into fluids flowing under a variety of conditions and identified the group of parameters now known as the Reynolds Number for determining whether pipe flow will be laminar or turbulent. He observed that under laminar flow conditions, the dye flows in a streamline and doesn't mix into the rest of the flowing fluid. Under turbulent flow conditions, the turbulence mixes dye into all of the flowing fluid. Based on Reynolds' experiments and subsequent measurements, the criterion now in widespread use is that **pipe flow will be laminar for a Reynolds Number (Re) less than 2100 and it will be turbulent for a Re**

greater than 4000. For 2100 < Re < 4000, called the transition region, the flow may be either laminar or turbulent, depending upon factors like the entrance conditions into the pipe and the roughness of the pipe surface. In general transition region conditions should be avoided in designing piping systems.

The Reynolds Number for flow in pipes is defined as: $\mathbf{Re = DV\rho/\mu}$, where

- **D** is the diameter of the pipe in ft (m for S.I.)
- **V** is the average fluid velocity in the pipe in ft/sec (m/s for S.I) (The definition of average velocity is: V = Q/A, where Q = volumetric flow rate and A = cross-sectional area of flow)
- ρ is the density of the fluid in slugs/ft^3 (kg/m^3 for S.I.)
- μ is the viscosity of the fluid in lb-sec/ft^2 (N-s/m^2 for S.I.)

Transport of natural gas in a pipeline is typically turbulent flow. Laminar flow typically takes place with liquids of high viscosity, like lubricating oils.

Example #6: Natural gas with a specific gravity of 0.65 is flowing at 60 cfs through a 12" diameter pipe with a gas temperature of 80 ^{o}F and average pressure of 500 psi. What is the Reynolds number of this flow? Is the flow laminar or turbulent?

Solution: From Example #5, the density and the viscosity of the natural gas are 1.765 lbm/ft^3 and 0.01175 cp. Converting to the required units for the Reynolds Number:

$$\rho_g = 1.765/32.17 = 0.05486 \text{ slugs/ft}^3$$

$$\mu_g = 0.01175*0.0000208854 = 2.45 \times 10^{-7} \text{ lb-sec/ft}^2.$$

The gas velocity, V, can be calculated from

$$V = Q/A = Q/(\pi D^2/4) = 60/[\pi(1)^2/4] = 76.4 \text{ ft/sec.}$$

Substituting values into $Re = DV\rho/\mu$ gives:
$Re = (1)(76.4)(0.05486)/(2.45 \times 10^{-7})$, or **$Re = 1.70 \times 10^7$**.

The Reynolds Number is much greater than 4000, so **the flow is turbulent**.

6. Options for Natural Gas Pipeline Flow Calculations

One possibility for natural gas flow calculations is the Darcy-Weisbach equation. This equation is for incompressible flow, and natural gas is clearly compressible, however, if the pressure drop is not too great, as detailed in the next section, the density change of the natural gas is small enough that the Darcy-Weisbach equation can be used.

If the pressure drop is too great to allow use of the Darcy Weisbach equation, then an equation such as the Weymouth Equation, the Panhandle A Equation, the Panhandle B Equation, the General Flow Equation, the IGT Equation, or the AGA Method should be used. The choice among these options depends upon factors such as pipe diameter, gas pressure, pipeline length, Reynolds number, and whether the flow can be considered isothermal or adiabatic.

More Details are provided in the following sections. Details of the Darcy Weisbach Equation, the Weymouth Equation, the Panhandle A Equation, and the Panhandle B Equation will be presented along with example calculations with each of those four equations.

7. The Darcy Weisbach Equation

This equation is for incompressible flow, where "incompressible" can be interpreted as "constant density." For natural gas flow in which the pressure drop is not too great, the density change of the gas is small enough that the Darcy-Weisbach equation can be used. According to Crane (1988), if the pressure drop is less than 10% of the inlet pressure, then for an incompressible flow calculation, the gas density may be calculated at either the upstream or the downstream pressure. If the pressure drop is between 10% and 40% of the upstream pressure, then the density of the gas should be calculated at the average of the upstream and downstream pressures for use in an incompressible flow equation such as the Darcy Weisbach equation.

The **Darcy Weisbach equation** is a widely used empirical relationship among several pipe flow variables. The equation is:

$$\mathbf{h_L = f(L/D)(V^2/2g)} \qquad (9)$$

where:

- **L** = pipe length, ft (m for S.I. units)
- **D** = pipe diameter, ft (m for S.I. units)
- **V** = average velocity of fluid (= **Q/A**), ft/sec (m/s for S.I. units)
- $\mathbf{h_L}$ **=** frictional head loss due to flow at an ave. velocity, **V**, through a pipe of diameter, **D**, and length, **L**, ft (ft-lb/lb) (m or N-m/N for S.I. units).
- **g** = acceleration due to gravity = 32.2 ft/sec^2 (9.81 m/s^2)
- **f** = Moody friction factor (a dimensionless empirical factor that is a function of Reynolds Number and ε/D, where:
- **ε** = an empirical pipe roughness, ft (m or mm for S.I. units)

NOTE: Although the Darcy Weisbach equation is an empirical equation, it is also a dimensionally consistent equation. That is, there are no dimensional constants in the equation, so any consistent set of units that gives the same dimensions on both sides of the equation can be used. A typical set of U.S. units and a typical set of S.I. units are shown in the list above.

Table 1, below, shows some typical values of pipe roughness for common pipe materials, drawn from information on several websites.

Table 1. Pipe Roughness Values

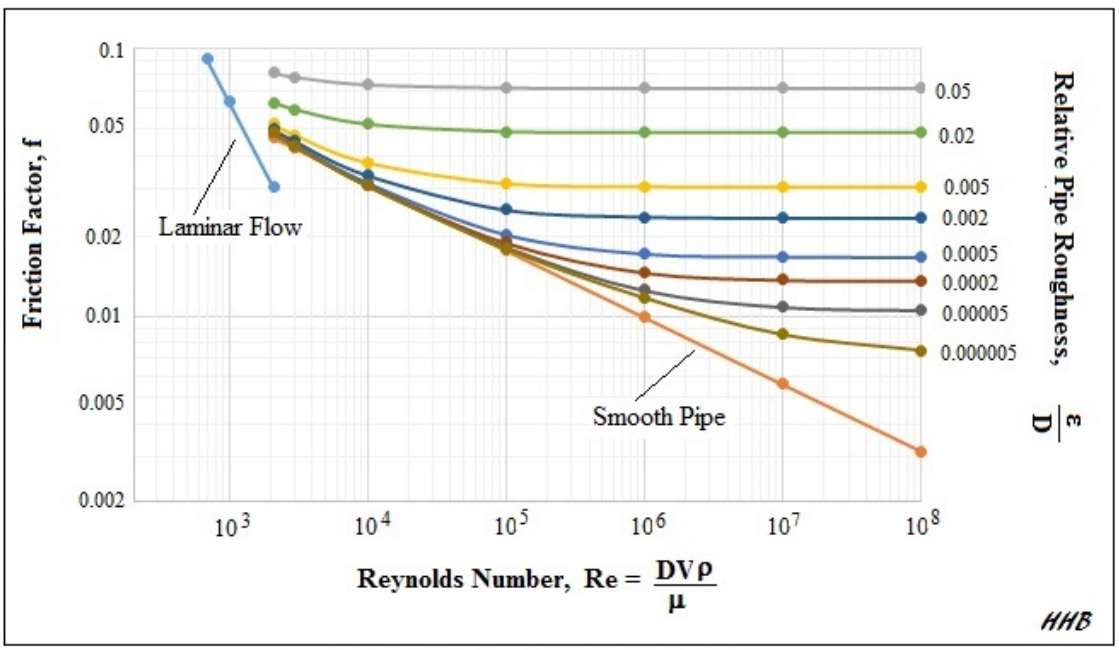
Pipe Material	Roughness, ε feet
drawn brass or copper	0.000005
PVC pipe	0.000005
commercial steel	0.000150
wrought iron	0.000150
asphalted cast iron	0.000400
galvanized iron	0.000500
cast iron	0.000850
concrete	0.001 - 0.01

Frictional pressure drop for pipe flow is related to the frictional head loss through the equation: $\Delta P_f = \rho g h_L = \gamma h_L$, where:

- h_L = frictional head loss (ft or m) as defined above
- ρ = fluid density, slugs/ft^3 (kg/m^3 for S.I.)
- g = acceleration due to gravity, ft/sec^2 (m/s^2 for S.I.)

- γ = specific weight, lb/ft^3 (N/m^3 for S.I.)

A value of the **Moody friction factor, f**, is needed for any calculations with the Darcy Weisbach equation other than empirical determination of the friction factor by measuring all of the other parameters in the equation. The friction factor, **f**, also appears in some of the other natural gas pipeline flow equations that will be presented shortly. One method of obtaining a value for **f** is graphically, from the Moody friction factor diagram, first presented by L. F. Moody in his classic 1944 paper in the *Transactions of the ASME*. (Ref. #1). The Moody friction factor diagram, shown in the diagram below, is now available in many handbooks and textbooks and on many websites.



Moody Friction Factor Diagram

There are also equations that may be use to calculate the Moody friction factor, f, rather than using a graph like the Moody diagram. There are, in fact, equations available that give the relationships between Moody friction factor and **Re & ε/D** for four different portions of the Moody diagram. The four portions of the Moody diagram are:

i) **laminar flow** (**Re** < 2100 – the straight line at the left side of the Moody diagram);

ii) **smooth pipe turbulent flow** - the dark curve labeled "smooth pipe" in the Moody diagram, in which the pipe roughness ratio, **ε/D**, is below the threshold value for a given value of **Re.** The friction factor, **f,** is a function of **Re** only in this region;

iii) **completely turbulent region** - the portion of the diagram above and to the right of the dashed line labeled "complete turbulence." The friction factor, **f,** is a function of **ε/D** only in this region;

iv) **transition region** - the portion of the diagram between the "smooth pipe" solid line and the "complete turbulence" dashed line. The friction factor, **f,** is a function of both **Re** and **ε/D** in this region.

A set of equations for these four regions are shown in the box below:

Laminar Flow: $$f = \frac{64}{Re}$$

Smooth Pipe Turbulent Flow: $$f = \frac{0.316}{Re^{1/4}}$$

Completely Turbulent Flow: $$f = \left[1.14 + 2\log_{10}\left(\frac{D}{\varepsilon}\right)\right]^{-2}$$

Transition Region: $$f = \left\{-2\log_{10}\left[\frac{(\varepsilon/D)}{3.7} + \frac{2.51}{Re\,(f^{1/2})}\right]\right\}^{-2}$$

Example #7: Calculate the value of the Moody friction factor for pipe flow with **Re** = 10^7 and **ε/D** = 0.005.

Solution: From the Moody diagram above, it is clear that the point, **Re** = 10^7 and **ε/D** = 0.005, is in the "complete turbulence" region of the diagram. Thus:

$$\mathbf{f = [1.14 + 2\log_{10}(D/\varepsilon)]^{-2}} = [1.14 + 2\log_{10}(1/0.005)]^{-2} = \underline{\mathbf{0.0303 = f}}$$

If the flow conditions are in the transition region of the Moody diagram, then the last equation shown above for **f**, often referred to as the Colebrook equation or the Colebrook-White equation (shown below), must be used. This equation cannot be solved explicitly for **f**, so an iterative solution is required as illustrated with the next example.

$$\mathbf{f = \{-2*\log_{10}[((\varepsilon/D)/3.7) + (2.51/(Re*(f^{1/2}))]\}^{-2}} \qquad (10)$$

Example #8: Calculate the value of the Moody friction factor for pipe flow with **Re** = 10^6 and **ε/D** = 0.0005.

Solution: The equation for the completely turbulent region, which is a function of **ε/D** only, will be used to obtain an initial estimate of **f**. Then an iterative calculation will be used to obtain a final value.

Initial estimate of **f** = $[1.14 + 2\log_{10}(1/0.0005)]^{-2} = 0.01668$

Substituting f = 0.01668 into the Colebrook equation gives:

$$\mathbf{f} = \{-2*\log_{10}[((.0005)/3.7) + (2.51/(10^6*(0.01668^{1/2}))]\}^{-2} = 0.01721$$

Repeating with the new value of f gives:

$$\mathbf{f} = \{-2*\log_{10}[((.0005)/3.7) + (2.51/(10^6*(0.01721^{1/2}))]\}^{-2} = 0.01721$$

Since the newly calculated value of f is the same as the previous estimate, this is the value of the friction factor:

<u>f = 0.01721</u>

Typical applications of the Darcy Weisbach equation include 1) calculation of frictional pressure drop or frictional head loss for specified pipe diameter, length, & material (roughness), and flow rate; 2) calculation of fluid flow rate for specified frictional pressure drop or frictional head loss along with pipe diameter, length & roughness; 3) calculation of required pipe diameter for specified frictional pressure drop or frictional head loss, fluid flow rate, and pipe length & roughness.

The general approach for calculation of flow rate (problem type (2) above) is to calculate the quantity $\mathbf{Re(f^{1/2})}$ with the following equation, derived from the Darcy Weisbach equation:

$$\mathbf{Re(f^{1/2}) = (\rho D/\mu)(2gh_L D/L)^{1/2}} \qquad (11)$$

Then the friction factor can be calculated with the Colebrook equation (equation (10)), the gas velocity can be calculated from the Darcy Weisbach equation, and the gas flow rate can be calculated from **Q = VA**.

Calculation of required pipe diameter (problem type (3) above) requires an iterative solution (in addition to the iterative determination of the friction factor).

The first two types of calculations will be illustrated with the next three examples.

Example #9: Determine the frictional pressure drop for the natural gas flow described in Example #5, that is, natural gas with a specific gravity of 0.65 flowing at 60 cfs through a 12" diameter pipe with a gas temperature of 80 ^{o}F and average pressure of 500 psi, if the pipe material is commercial steel and it is 500 ft long.

Solution: From Example #6, the gas density, viscosity, and velocity, and the Reynolds number are as follow:

ρ_g = 0.05486 slugs/ft^3, μ_g = 2.45 x 10^{-7} lb-sec/ft^2,

V = 76.4 ft/sec, and **Re =** 1.71 x 10^7.

From Table 1, the surface roughness for commercial steel pipe is
ε = 0.00015 ft, so **ε/D** = 0.00015/1 = 0.00015.

Substituting **Re =** 1.70 x 10^7 and **ε/D** = 0.00015 into the equation above for completely turbulent flow gives an initial estimate of **f** = 0.01295. Using this value of f along with the values for **Re** and **ε/D** in the Colebrook equation gives **f** = 0.013. Repeating gives the same value for **f**, so the friction factor can be taken as **f** = 0.013.

Now, substituting values into the Darcy Weisbach equation (equation (9)) gives:

$\mathbf{h_L = f(L/D)(V^2/2g)}$ = (0.013)(500/1)*[76.4^2/(2*32.17)] = 591 ft

The frictional pressure drop can then be calculated as follows:

$\mathbf{\Delta P_f = \rho g h_L}$ = (0.05486)(32.17)(591) = 1043 psf = 1043/144 psi = **<u>7.24 psi</u>**

Since the pressure drop is less than 10% of the pipe pressure, the incompressible calculation with the Darcy Weisbach equation is appropriate.

Example #10: If the length of the pipeline in Example #9 was 20,000 ft instead of 500 ft, would the Darcy Weisbach equation still be useable to calculate the frictional pressure drop?

Solution: Following the same calculation procedure used for Example #9 leads to a frictional pressure drop of 290 psi. This is more than 40% of the average pipe pressure, so the Darcy Weisbach equation should not be used.

Example #11: Determine the flow rate of natural gas through a 12 inch diameter, 500 ft long pipeline of commercial steel, if the specific gravity of

the gas is 0.65, its temperature is 80 °F, its average pressure is 500 psia, and the pressure drop across the 500 ft of pipe is 20 psi.

Solution: From Example #5, the gas density, viscosity are as follow:

$$\rho_g = 0.05486 \text{ slugs/ft}^3, \quad \mu_g = 2.45 \times 10^{-7} \text{ lb-sec/ft}^2,$$

From Table 1, the surface roughness for commercial steel pipe is $\varepsilon = 0.00015$ ft, so $\boldsymbol{\varepsilon/D} = 0.00015/1 = 0.00015$.

The frictional head loss is:

$$\mathbf{h_L = \Delta P_f / \rho g} = (20*144)/(0.05486*32.17) = 1632 \text{ ft}$$

Using equation (11):

$$\mathbf{Re(f^{1/2})} = [0.05486*1/(2.45 \times 10^{-7})](2*32.17*1632*1/500)^{1/2} = 3{,}244{,}930$$

Using the Colebrook equation:

$$\mathbf{f} = \{-2*\log_{10}[((0.00015/1)/3.7) + (2.51/(3244930))]\}^{-2} = \mathbf{0.0130}$$

Now, from the Darcy Weisbach equation, the gas velocity is:

$$V = [2gh_LD/(fL)]^{1/2} = [2*32.17*1632/(0.0130*500)]^{1/2} = 127.1 \text{ ft/sec}$$

Finally, $\mathbf{Q = VA = V(\pi D^2/4)} = 127.1(\pi 1^2/4) =$ **<u>99.8 cfs</u>**

The flow rate of natural gas is often expressed as MMSCFD (millions of standard cubic feet per day). The calculation in Example #11 gives actual cfs (ACFS). The flow rate in ACFS can be converted to SCFS (standard cfs) with the following equation:

$$\mathbf{SCFS = ACFS(P_{ave}/P_b)(T_b/T_{ave})}$$

Where: **SCFS** is the flow rate in standard cubic feet per second.

ACFS is the flow rate in actual cubic feet per second.

$\mathbf{P_{ave}}$ is the average absolute pressure of the gas.

$\mathbf{P_b}$ is the absolute base pressure (standard pressure) expressed in the same units as $\mathbf{P_{ave}}$. ($\mathbf{P_b}$ is typically 14.7 psia.)

$\mathbf{T_{ave}}$ is the average absolute temperature of the gas in ^oR.

$\mathbf{T_b}$ is the absolute base temperature (standard temperature) in ^oR. ($\mathbf{T_b}$ is typically 60 ^oF = 520 ^oR.)

Multiplying SCFS by 3600*24 sec/day and dividing by 10^6, will convert it to MMSCFD (millions of standard cubic feet per day).

Example #12: Convert the natural gas flow rate calculated in Example #11 (97.9 cfs) into units of MMCFD (millions of cubic feet per day) and into MMSCFD (millions of standard cubic feet per day).

Solution: The conversion can be made as follows:

Q = (99.8 ft^3/sec)(3600 sec/hr)(24 hr/day)/(10^6 ft^3/MMCF)

= **<u>8.62 MMCFD</u>**

This can be converted to MMSCFD by the following calculation:

Q = 8.62 MMCFD(500/14.7)[(60 + 460)/(80+460)] = **<u>282 MMSCFD</u>**

8. The Weymouth Equation

The Weymouth equation was the first natural gas pipeline flow equation that was developed to make calculations without the requirement for an iterative calculation to obtain a value for the friction factor, f. The Weymouth equation is most suitable for gas flows in pipelines with diameters of 15 inches or less, for segments of pipeline that are less than 20 miles in length, and for medium to high pressure (approximately 100 psia to 1000 psia). The flow is typically considered to be isothermal when using the Weymouth equation.

A commonly used form of the Weymouth equation in U.S. units, which takes elevation difference between pipeline entrance and exit into account, is as follows:

$$Q = 433.49\, E \left(\frac{T_b}{P_b}\right)\left(\frac{P_1^2 - e^s P_2^2}{G T_f L_e Z}\right)^{0.5} D^{8/3} \qquad (12)$$

Where:

$$L_e = \frac{L\,(e^s - 1)}{s}$$

$$s = 0.0375\, G \left(\frac{\Delta H}{T_f\, Z}\right)$$

The parameters in the equation and their units are as follows:

Q is the natural gas flowrate in std ft^3/day.
(Dividing this value by 10^6 gives MMSCFD.)

T_b is the base temperature, aka temperature at standard conditions, in ^oR.

(The base temperature, **T_b**, is typically 60 ^oF = 520 ^oR.)

P_b is the base pressure, aka pressure at standard conditions, in psia.
(The base pressure, **P_b**, is typically 14.7 psia.)

P_1 is the pressure at the pipe entrance in psia.

$\mathbf{P_2}$ is the pressure at the pipe exit in psia.

L is the length of the pipeline in miles.

$\mathbf{L_e}$ is the effective length of the pipeline in miles, taking into account the elevation difference between pipeline entrance and exit. (See the equation above for calculation of $\mathbf{L_e}$.)

D is the internal diameter of the pipe in inches.

G is the specific gravity of the natural gas relative to air.

$\mathbf{T_f}$ is the average temperature of the gas flowing in the pipeline in ^{o}R.

Z is the compressibility factor of the natural gas at the average temperature and pressure of the gas in the pipeline.

ΔH is the elevation of the pipeline exit minus the elevation of the pipeline entrance in ft.

NOTE: The equation for the effective length of the pipe, $\mathbf{L_e}$, will work for either a positive or negative value of **ΔH**, but it will not work if **ΔH** is set equal to zero because there will be division by zero in the equation. If there is no elevation difference between the pipeline entrance and exit, just set **ΔH** equal to 1 ft. If the elevation increases in the direction of flow, **ΔH** should have a positive value and if the elevation decreases in the direction of flow, **ΔH** should have a negative value.

E is the pipeline efficiency.

The value of **E** may range from 0.85 to 1. Following are some general guidelines for the value of the pipeline efficiency, **E**:

1. **E** = 0.92 for average operating conditions.

2. **E** = 1 for new pipe with no bends, fittings or diameter changes.

3. **E** = 0.95 for very good operating conditions, typically through the first 12-28 months of operation.

4. **E** = 0.85 for unfavorable operating conditions.

Example calculations with the Weymouth Equation are included in Section 11 below.

9. The Panhandle A Equation

The Panhandle A equation was developed in the 1940s as a variation of the General Flow Equation. The Panhandle A equation is most suitable for gas flows in long pipelines with diameters of 12 to 60 inches and pressures between 800 psia and 1500 psia. The flow is typically considered to be isothermal when using the Panhandle A equation.

A commonly used form of the Panhandle A equation in U.S. units, which takes elevation difference between pipeline entrance and exit into account, is as follows:

$$Q = 435.87\, E \left(\frac{T_b}{P_b}\right)^{1.0788} \left(\frac{P_1^2 - e^s P_2^2}{G^{0.8539}\, T_f L_e Z}\right)^{0.5394} D^{2.6182} \tag{13}$$

Where:

$$L_e = \frac{L\,(e^s - 1)}{s}$$

$$s = 0.0375\, G \left(\frac{\Delta H}{T_f\, Z}\right)$$

The parameters in this equation are all the same as those just listed above for the Weymouth Equation.

Example calculations with the Panhandle A Equation are included in Section 11 below.

10. The Panhandle B Equation

The Panhandle B equation was developed in the 1940s as a variation of the General Flow Equation. The Panhandle B equation is most suitable for gas flows in long pipelines with diameters of 36 inches or larger and pressures above 1000 psia. The flow is typically considered to be isothermal when using the Panhandle B equation.

A commonly used form of the Panhandle B equation in U.S. units, which takes elevation difference between pipeline entrance and exit into account, is as follows:

$$Q = 737\, E \left(\frac{T_b}{P_b}\right)^{1.02} \left(\frac{P_1^2 - e^s P_2^2}{G^{0.961}\, T_f L_e Z}\right)^{0.51} D^{2.53} \qquad (14)$$

Where:

$$L_e = \frac{L\,(e^s - 1)}{s}$$

$$s = 0.0375\, G \left(\frac{\Delta H}{T_f\, Z}\right)$$

The parameters in this equation are all the same as those listed above for the Weymouth Equation.

Example calculations with the Panhandle B Equation are included in Section 11 below.

11. Example Calculations

This section consists of an example with calculations using the Weymouth Equation, the Panhandle A Equation, and the Panhandle B Equation.

Example #13: Use the Weymouth Equation, the Panhandle A Equation, and the Panhandle B Equation to calculate the natural gas flow rate for the gas and pipeline information in Example #11. Assume no elevation difference between pipeline entrance and exit, and assume a pipeline efficiency of 0.92. Compare with the result calculated using the Darcy Weisbach Equation in Example #11.

Solution: Here is the information from Example #11: "Determine the flow rate of natural gas through a 12 inch diameter, 500 ft long pipeline of commercial steel, if the specific gravity of the gas is 0.65, its temperature is 80 °F, its average pressure is 500 psia, and the pressure drop across the 500 ft of pipe is 20 psi."

The base temperature and pressure will be taken as $\mathbf{T_b}$ = 60 °F = 520 °R and $\mathbf{P_b}$ = 14.7 psia. Other given parameters needed for the calculations are: pipe diameter, **D** = 12 in; pipe length, **L** = 500/5280 mi = 0.094697 mi; gas specific gravity, **G** = 0.65; flowing temperature, $\mathbf{T_f}$ = 80 °F; pipeline efficiency, **E** = 0.92. For no elevation change between pipeline entrance and exit, set **ΔH** = 1 ft.

Intermediate calculations are as follows:

From Equation (6): Z = 0.919

From the equations below the Weymouth, Panhandle A, and Panhandle B Equations: **s** = 0.00005 and $\mathbf{L_e}$ = 0.0947 mi.

The inlet and exit pressures are calculated from the average pressure and the pressure drop as follows:

$$\mathbf{P_1} = \mathbf{P_{ave}} + (\Delta\mathbf{P}/2) = 500 + 20/2 = 510 \text{ psia}$$

$\mathbf{P_2} = \mathbf{P_{ave}} - (\Delta\mathbf{P}/2) = 500 - 20/2 = 490$ psia

Substituting these values into Equation (11) (the Weymouth Equation) gives:

$\mathbf{Q} = 2.72 \times 10^8$ SCFD = **<u>272 MMSCFD</u>** (Weymouth Equation)

Substituting the same input values into Equation (12) (the Panhandle A Equation) gives:

$\mathbf{Q} = 4.01 \times 10^8$ SCFD = **<u>401 MMSCFD</u>** (Panhandle A Equation)

Substituting the same input values into Equation (13) (the Panhandle B Equation) gives:

$\mathbf{Q} = 3.74 \times 10^8$ SCFD = **<u>375 MMSCFD</u>** (Panhandle B Equation)

From Example #12, the results using the Darcy Weisbach Equation were:

Q = **<u>282 MMSCFD</u>**

Discussion: Note that the pipe diameter and length and the pipeline pressure are all within the recommended ranges for use of the Weymouth Equation. Also, the pressure drop is less than 10 % of the average pipeline pressure, so use of the Darcy Weisbach Equation is appropriate. The gas flow rates calculated with these two equations are in rather close agreement. On the other hand, the pipe diameter and length and the pipe line pressure are not within the recommended ranges for use of either the Panhandle A Equation or the Panhandle B Equation and both of those equations yielded gas flow rates quite a bit higher than those calculated with the Weymouth and Darcy Weisbach Equations.

12. Summary

The Weymouth Equation, the Panhandle A Equation, the Panhandle B Equation, and the Darcy Weisbach Equation are all possible equations to be used for natural gas pipeline flow calculations. These four equations were presented, discussed, and illustrated with numerous worked examples in this course material. Several natural gas properties used in the calculations were also identified and discussed, along with presentation of methods for calculating values for some of the properties.

13. References and Websites

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